Phenomenological description and future scenarios of spread of Covid-19 infection in Italy

arxiv.org/abs/2011.08111

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Submitted to “Scientific Reports”

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Outline

Experimental observations: correlation between number of people tested positive for the virus and number of hospitalized cases

Compartmental model: how we can evaluate people mobility by analyzing the time dependence of hospitalized cases

Pre–season holidays behavior: variable restriction measures vs general lockdown

Consequence of the mobility increase during season holidays

Effects of vaccination and future trends

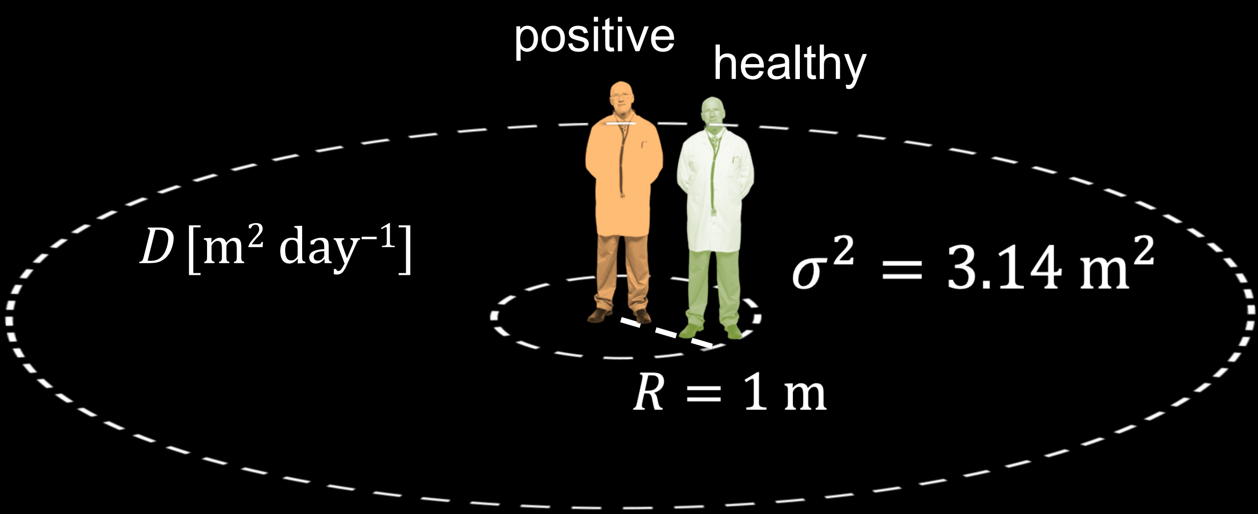
Modelling the impact of virus variants

Compartmental model

$$\frac{dp}{dt} = \rho D \sigma^2 (\rho - c) p - \frac{dg}{dt} - \frac{dm}{dt}$$

p	active positives
g	healed people
m	fatalities
c	people who have contracted the virus
ρ	inhabitant density
D	diffusion coefficient
σ^2	infection cross–section

$\sigma^2 = \pi R^2$ The cross–section σ^2 measures the probability of a single infection event



D is the average surface area covered by a single person per day

ρD is the average number of people within the travel radius of a single person per day

Compartmental model

$$\frac{dp}{dt} = \rho D\sigma^2(\rho - c)p - \frac{dg}{dt} - \frac{dm}{dt}$$

p active positives

g healed people

m fatalities

c people who contracted the virus

ρ inhabitant density

D diffusion coefficient

σ^2 infection cross–section

We include two further compartments

$$\boxed{\tau_1 \quad f \quad q \quad D\sigma^2}$$

r hospitalized people

$$\frac{dr}{dt} = f\rho D\sigma^2(\rho - c)p - \left(\frac{q}{\tau_1} + \frac{1-q}{\tau_2}\right)r$$

$$\tau_2 = 20 \text{ days}$$

$$\tau_3 = 14 \text{ days}^*$$

f fraction of the new active positives that requires hospitalization

q fraction of the hospitalized people that dies in a characteristic time τ_1

$1-q$ fraction of the hospitalized people that heals in a characteristic time τ_2

s people who do not exhibit serious symptoms until complete healing

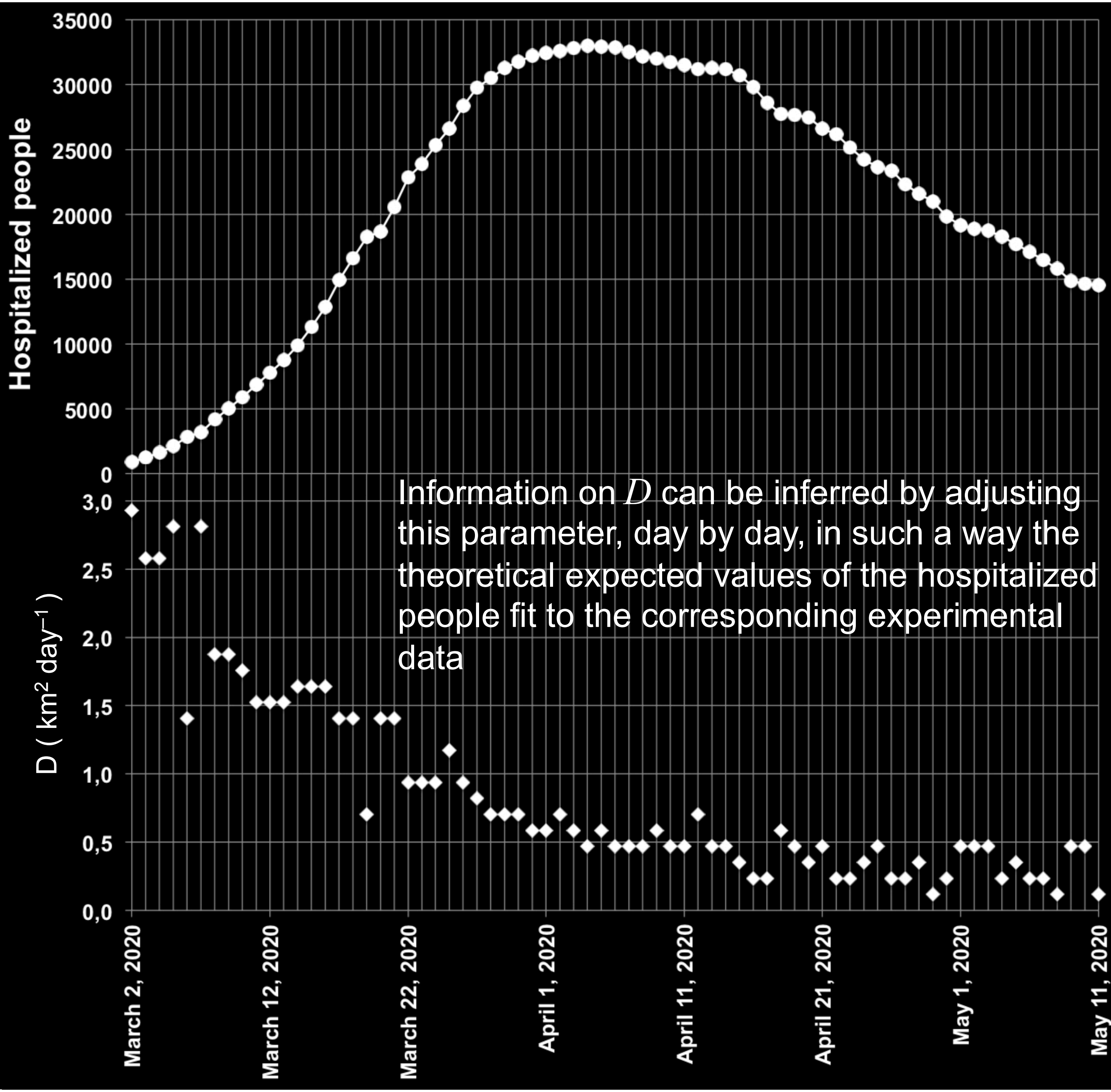
$$\frac{ds}{dt} = (1-f)\rho D\sigma^2(\rho - c)p - \frac{s}{\tau_3}$$

τ_3 characteristic healing time of infected people without serious symptoms

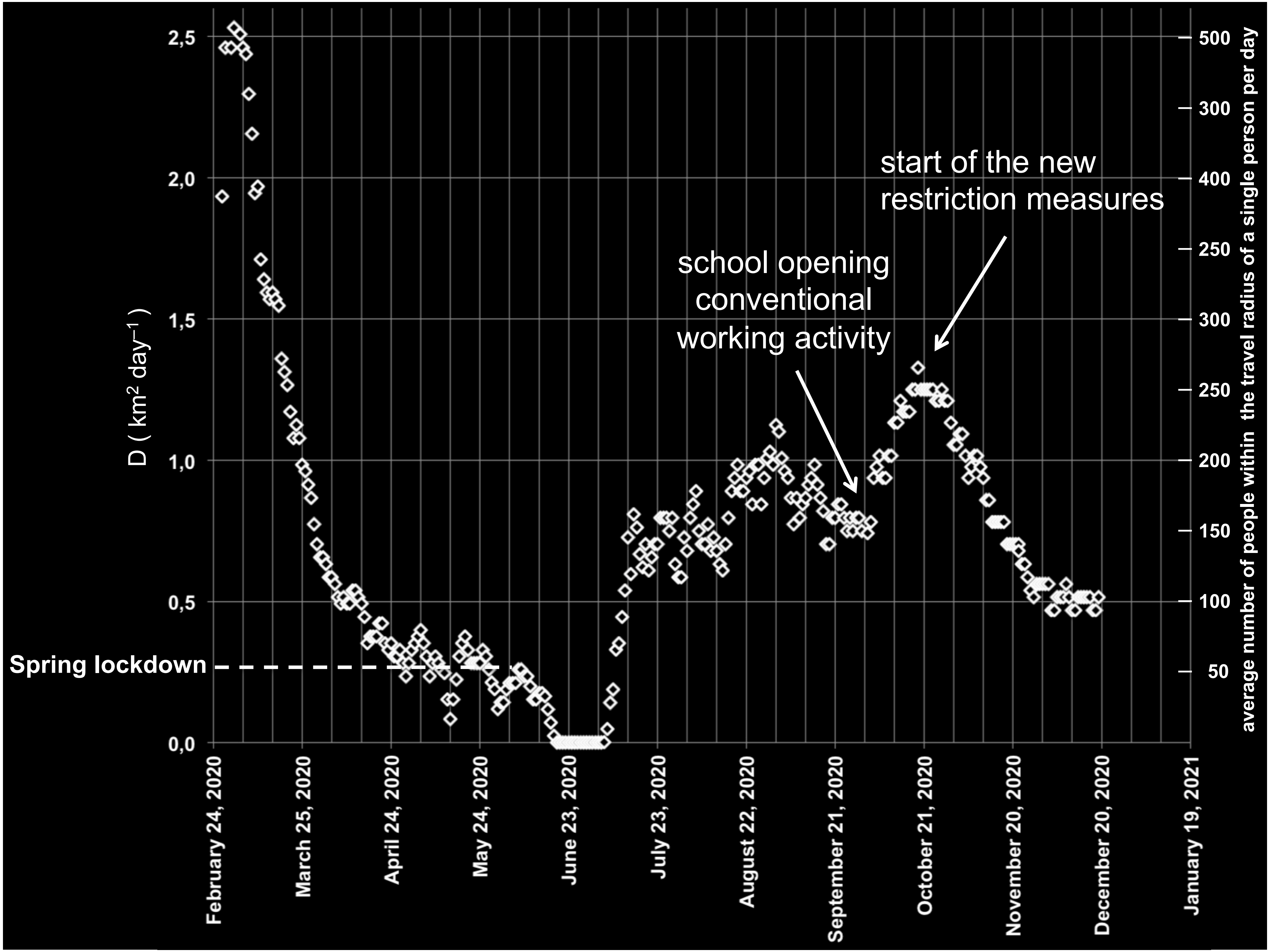
$$\frac{dm}{dt} = \frac{q}{\tau_1}r \qquad \frac{dg}{dt} = \frac{s}{\tau_3} + \frac{(1-q)r}{\tau_2} \qquad \frac{dc}{dt} = \rho D\sigma^2(\rho - c)p$$

* G. Morone et al. Frontiers in Medicine 7, Art. No. 562 (2020)

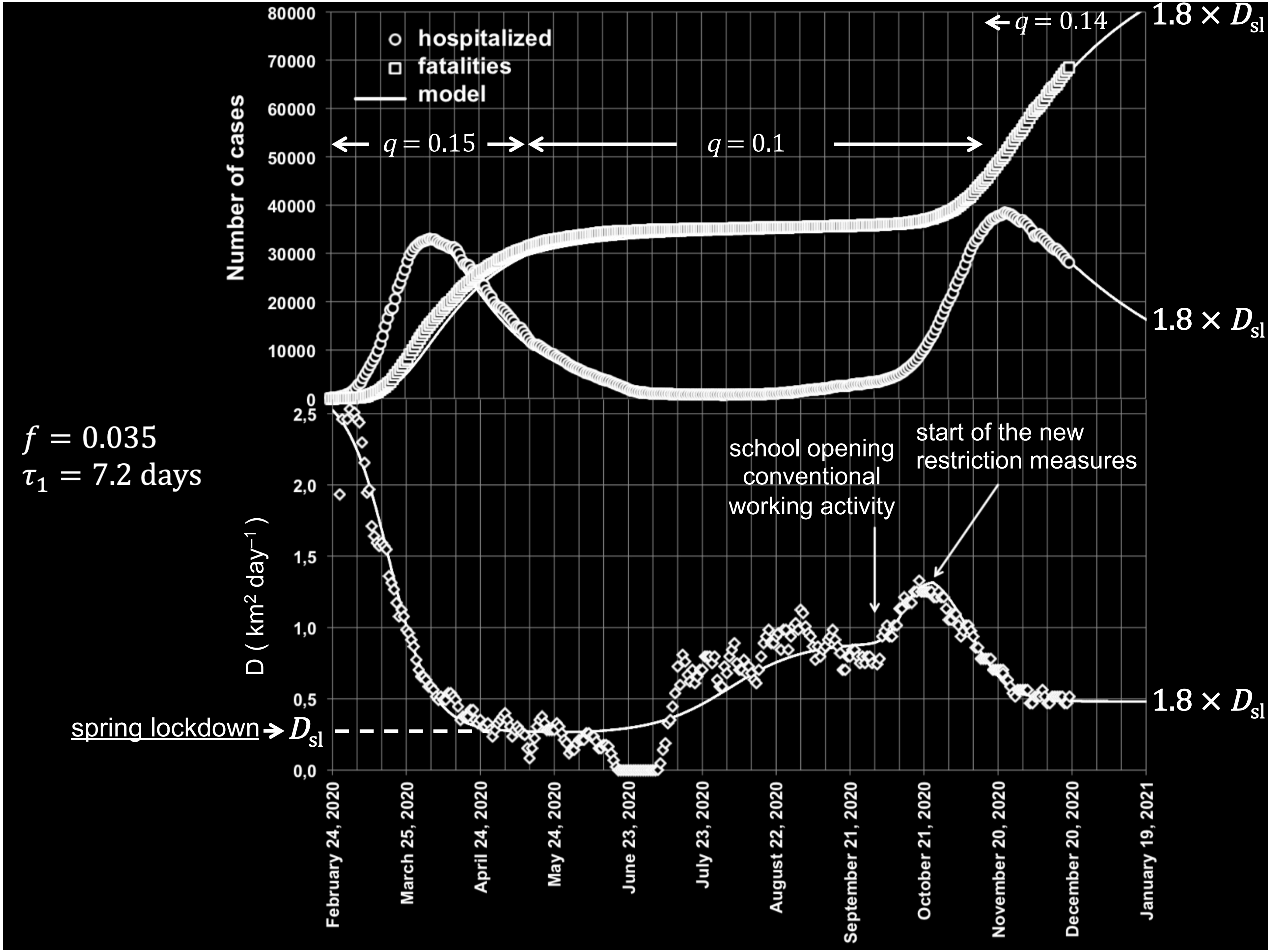
$f = 0.035$
$q = 0.15$
$\tau_1 = 7.2$ days



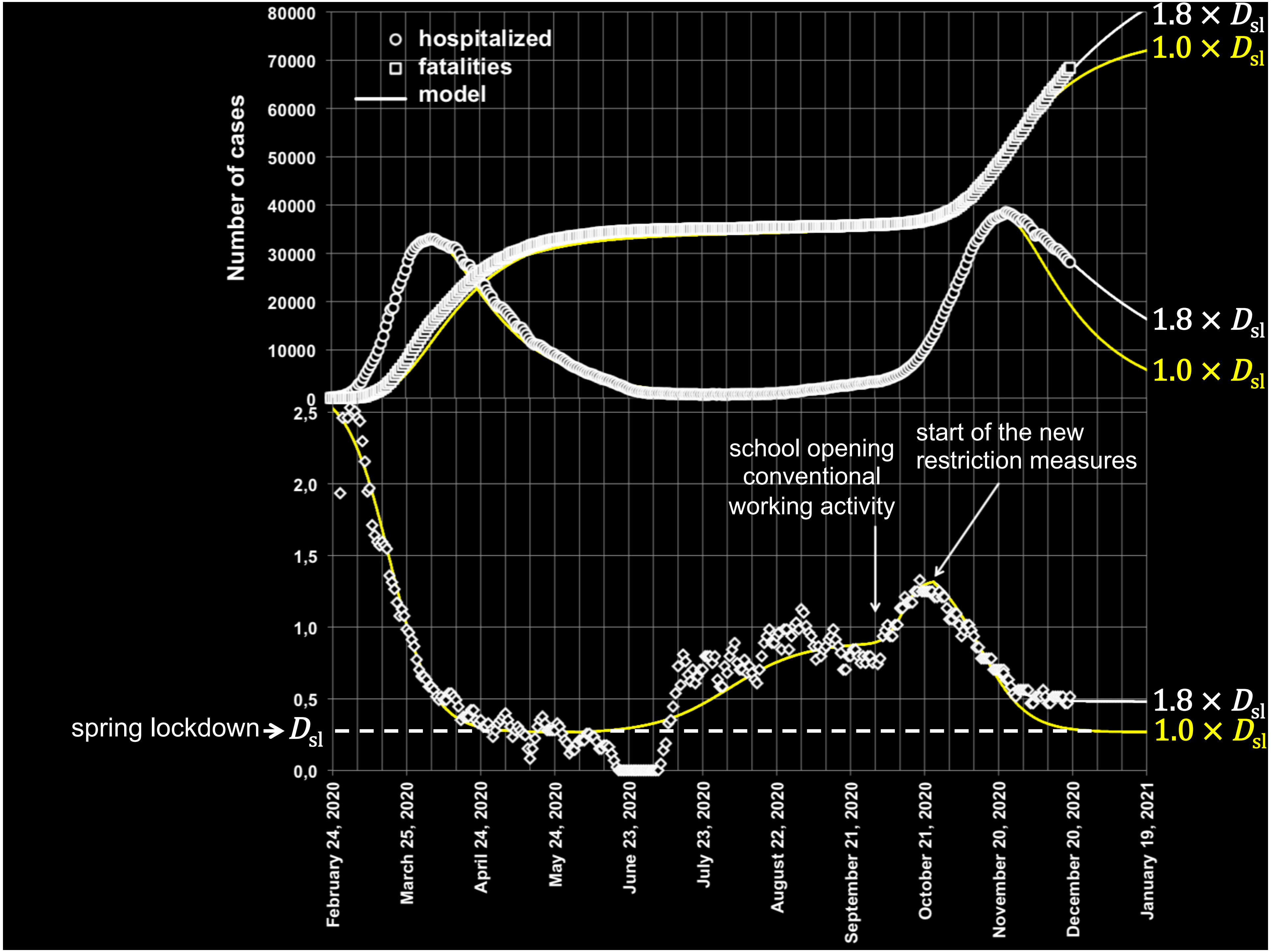
Information on D can be inferred by adjusting this parameter, day by day, in such a way the theoretical expected values of the hospitalized people fit to the corresponding experimental data



D (km² day⁻¹)
2,5
2,0
1,5
1,0
0,5
0,0
Spring lockdown
school opening
conventional
working activity
start of the new
restriction measures
February 24, 2020
March 25, 2020
April 24, 2020
May 24, 2020
June 23, 2020
July 23, 2020
August 22, 2020
September 21, 2020
October 21, 2020
November 20, 2020
December 20, 2020
January 19, 2021
500
300
400
250
300
250
200
150
100
50
average number of people within the travel radius of a single person per day



hospitalized
fatalities
model
Number of cases
80000
70000
60000
50000
40000
30000
20000
10000
0
$q = 0.15$
$q = 0.1$
$q = 0.14$
$1.8 \times D_{sl}$
$1.8 \times D_{sl}$
$f = 0.035$
$\tau_1 = 7.2$ days
D (km^2 day^{-1})
2,5
2,0
1,5
1,0
0,5
0,0
school opening
conventional
working activity
start of the new
restriction measures
spring lockdown D_{sl}
$1.8 \times D_{sl}$
February 24, 2020
March 25, 2020
April 24, 2020
May 24, 2020
June 23, 2020
July 23, 2020
August 22, 2020
September 21, 2020
October 21, 2020
November 20, 2020
December 20, 2020
January 19, 2021



hospitalized
fatalities
model
Number of cases
80000
70000
60000
50000
40000
30000
20000
10000
0
2,5
2,0
1,5
1,0
0,5
0,0
school opening
conventional
working activity
start of the new
restriction measures
spring lockdown → D_{sl}
$1.8 \times D_{sl}$
$1.0 \times D_{sl}$
$1.8 \times D_{sl}$
$1.0 \times D_{sl}$
$1.8 \times D_{sl}$
$1.0 \times D_{sl}$
February 24, 2020
March 25, 2020
April 24, 2020
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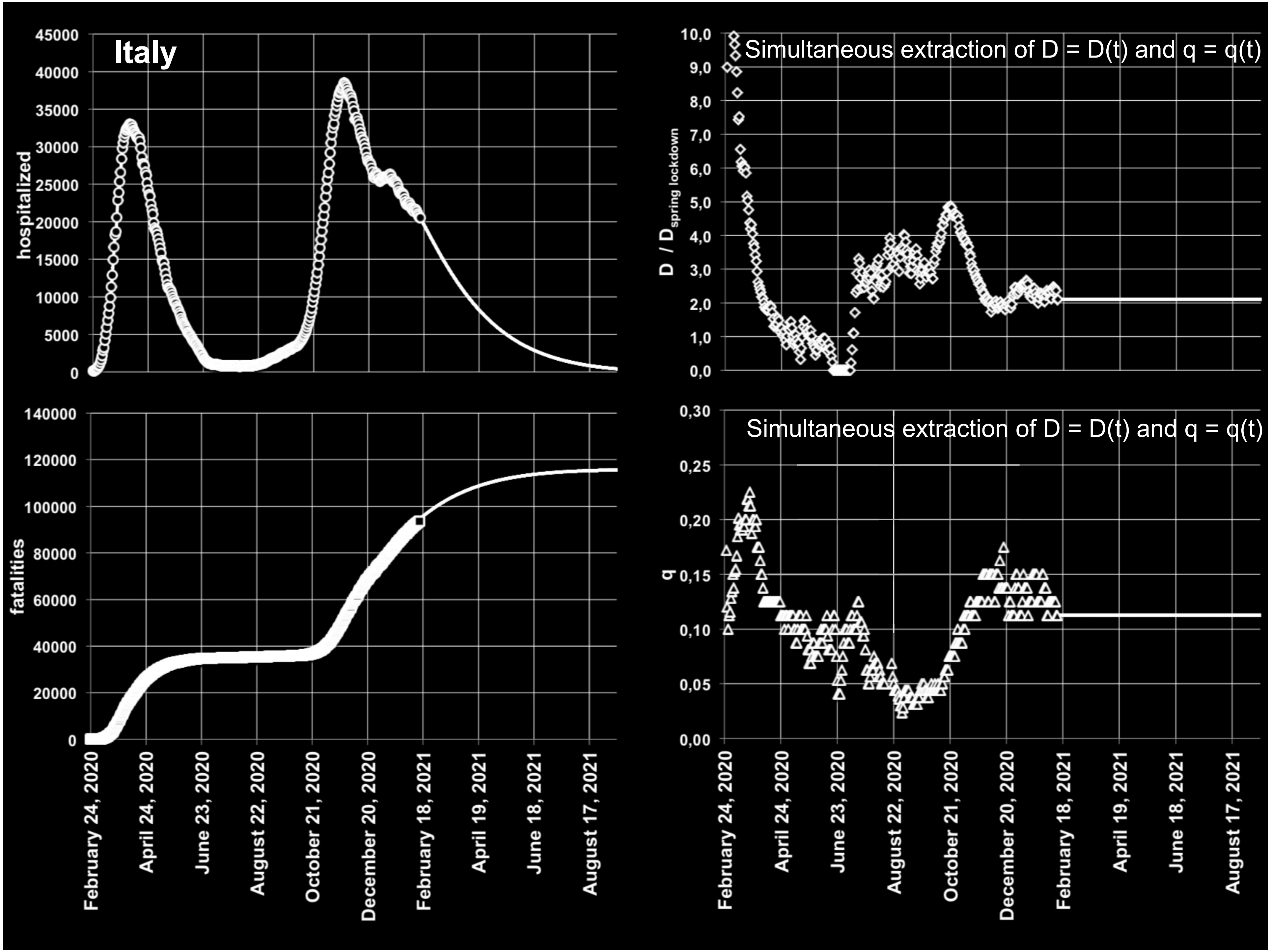
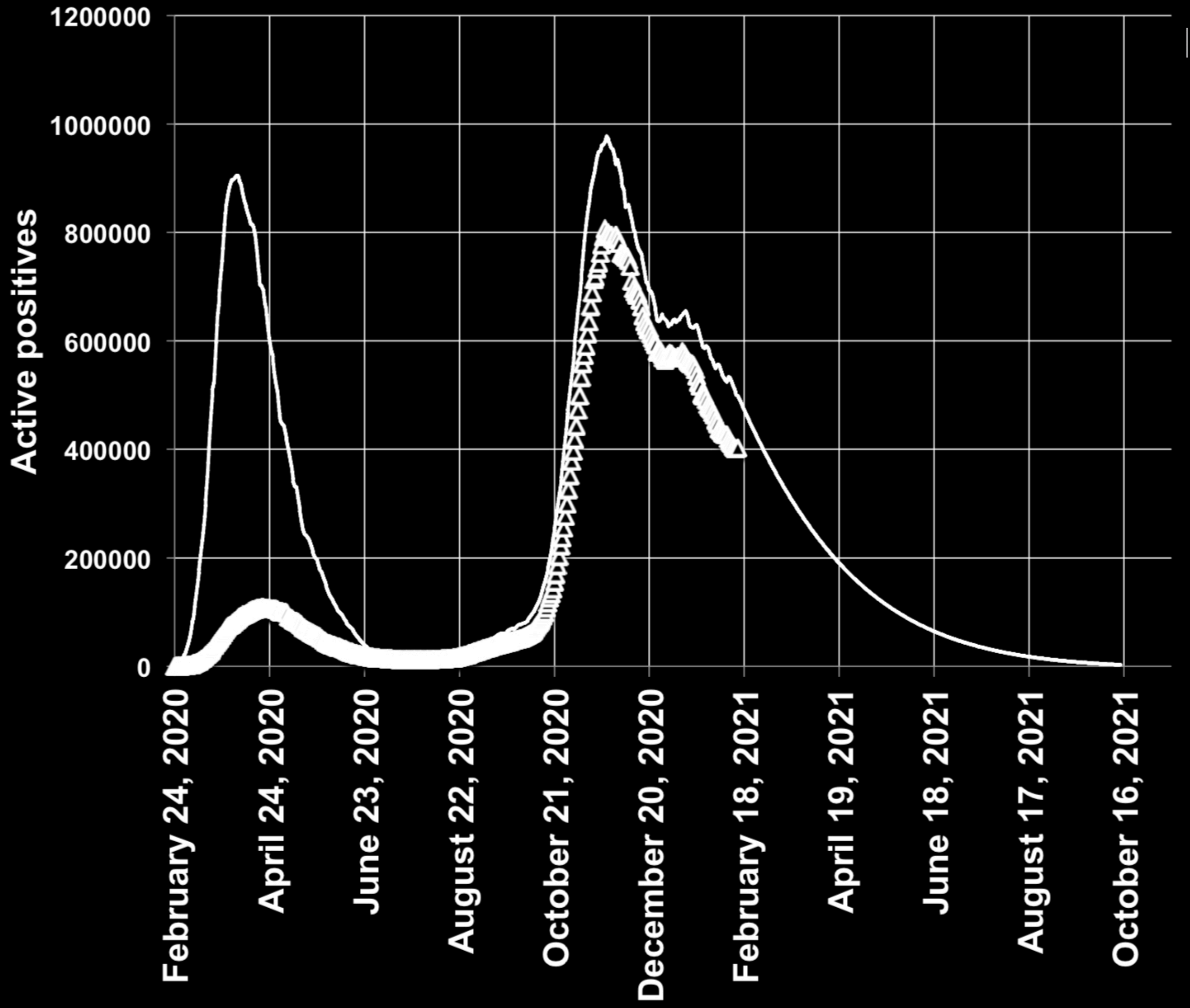
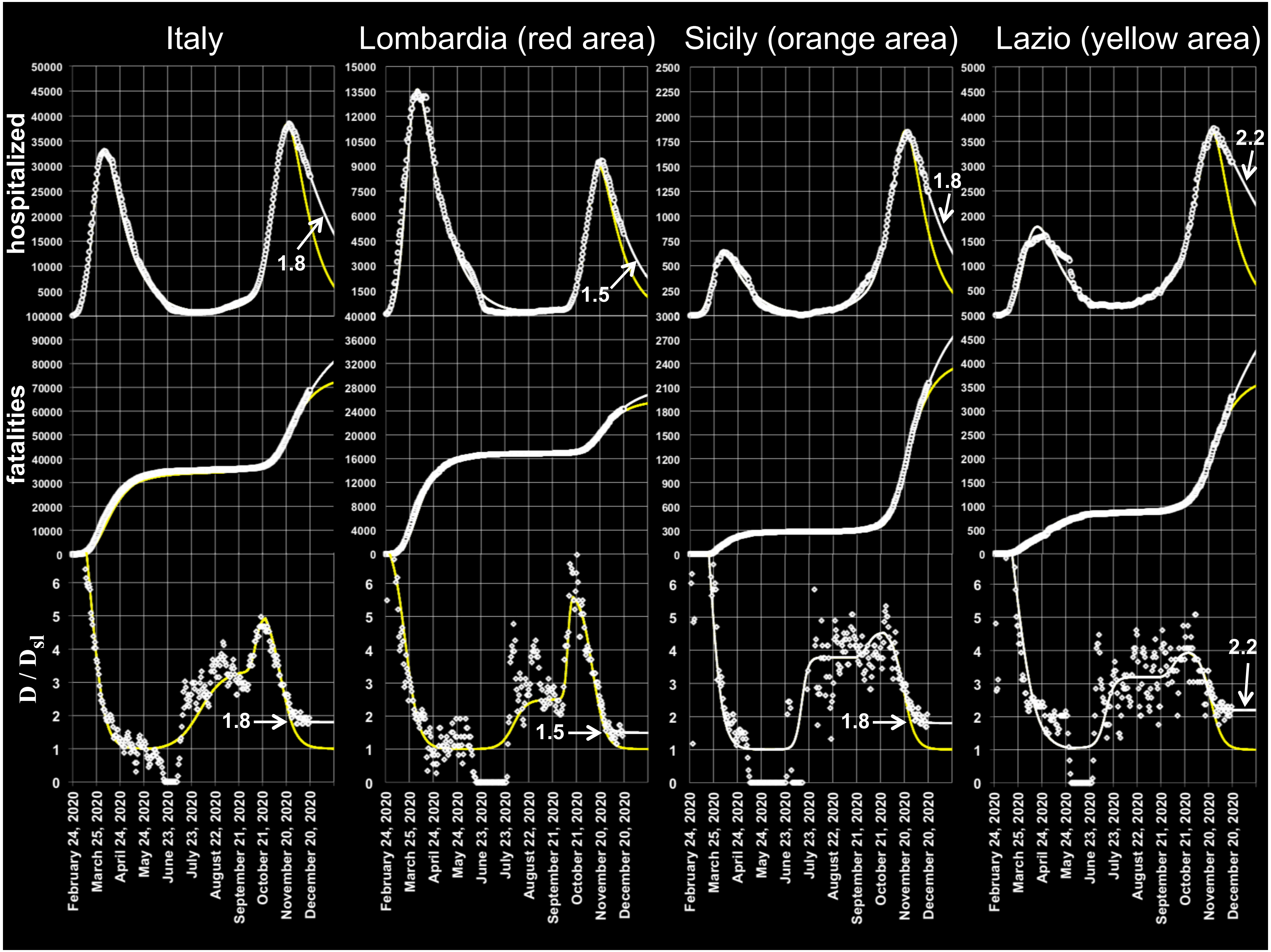
June 23, 2020
July 23, 2020
August 22, 2020
September 21, 2020
October 21, 2020
November 20, 2020
December 20, 2020
January 19, 2021


Italy
hospitalized
fatalities
Simultaneous extraction of D = D(t) and q = q(t)
D / D spring lockdown
q
Simultaneous extraction of D = D(t) and q = q(t)
February 24, 2020
April 24, 2020
June 23, 2020
August 22, 2020
October 21, 2020
December 20, 2020
February 18, 2021
April 19, 2021
June 18, 2021
August 17, 2021

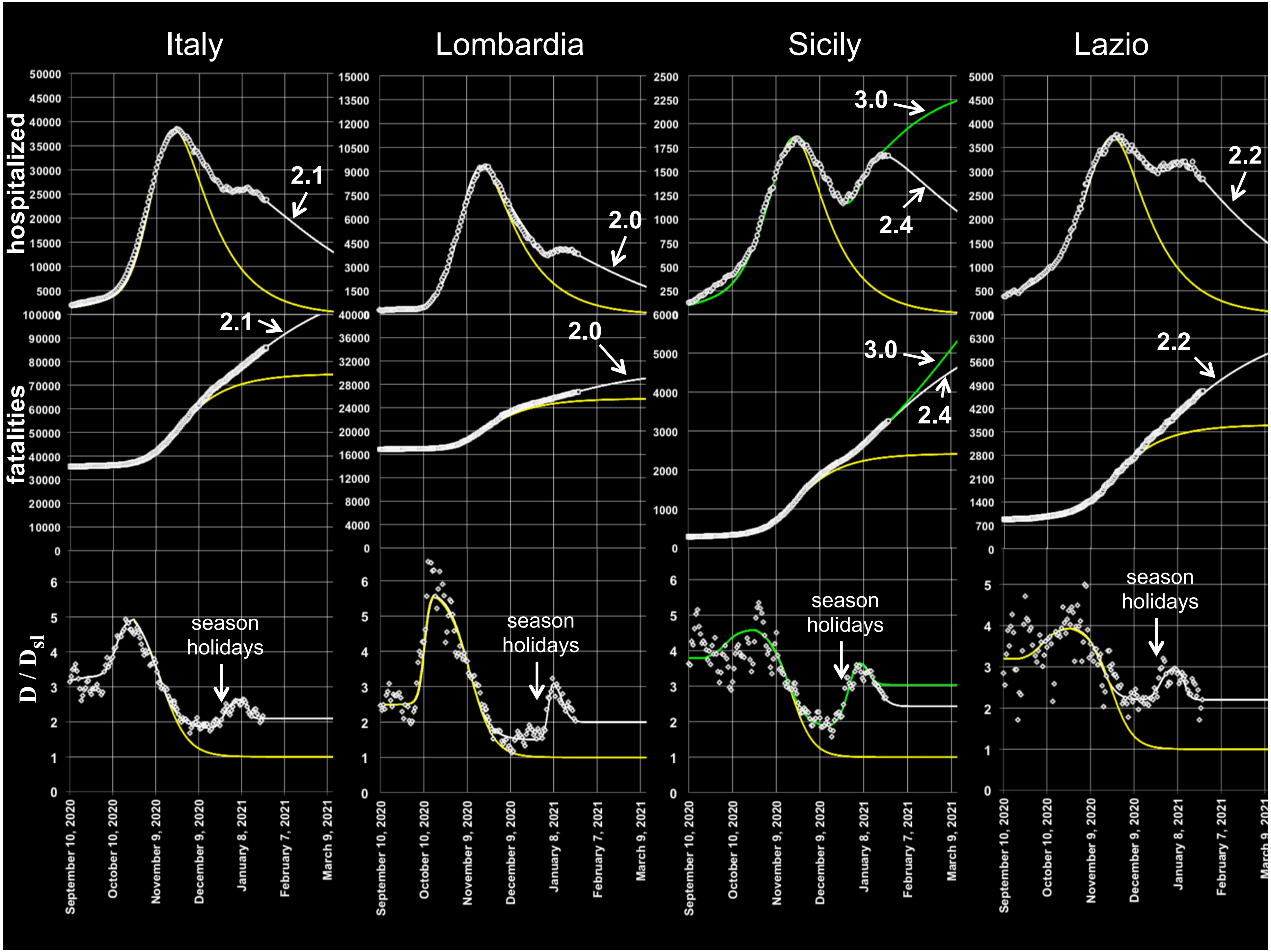
The simulated number of active positives is larger than the number of those tested positives for the virus: a fraction of circulating positives are not revealed by the testing procedure





Italy
Lombardia (red area)
Sicily (orange area)
Lazio (yellow area)
hospitalized
fatalities
D / D$_{sl}$
1.8
1.5
1.8
2.2
1.8
1.5
1.8
2.2
February 24, 2020
March 25, 2020
April 24, 2020
May 24, 2020
June 23, 2020
July 23, 2020
August 22, 2020
September 21, 2020
October 21, 2020
November 20, 2020
December 20, 2020

Consequences of the increasing mobility during season holydays



Italy
Lombardia
Sicily
Lazio
hospitalized
fatalities
D / D$_{sl}$
2.1
2.1
2.0
2.0
3.0
2.4
3.0
2.4
2.2
2.2
season holidays
season holidays
season holidays
season holidays
September 10, 2020
October 10, 2020
November 9, 2020
December 9, 2020
January 8, 2021
February 7, 2021
March 9, 2021

Effects of the vaccination

$$\rho = \rho_0 \qquad t < t_0$$

$$\rho = \rho_0 - \int_{t_0+\tau}^{t} v(t-\tau)dt \qquad t \geq t_0 + \tau$$

$$dc = \rho_0 D \sigma^2 [\rho - c] p$$

$\tau = 1$ month is the time interval for the vaccine immunization

t_0 is the day at which the vaccination campaign started (Dec. 31, 2020)

ρ_0 is the number inhabitants

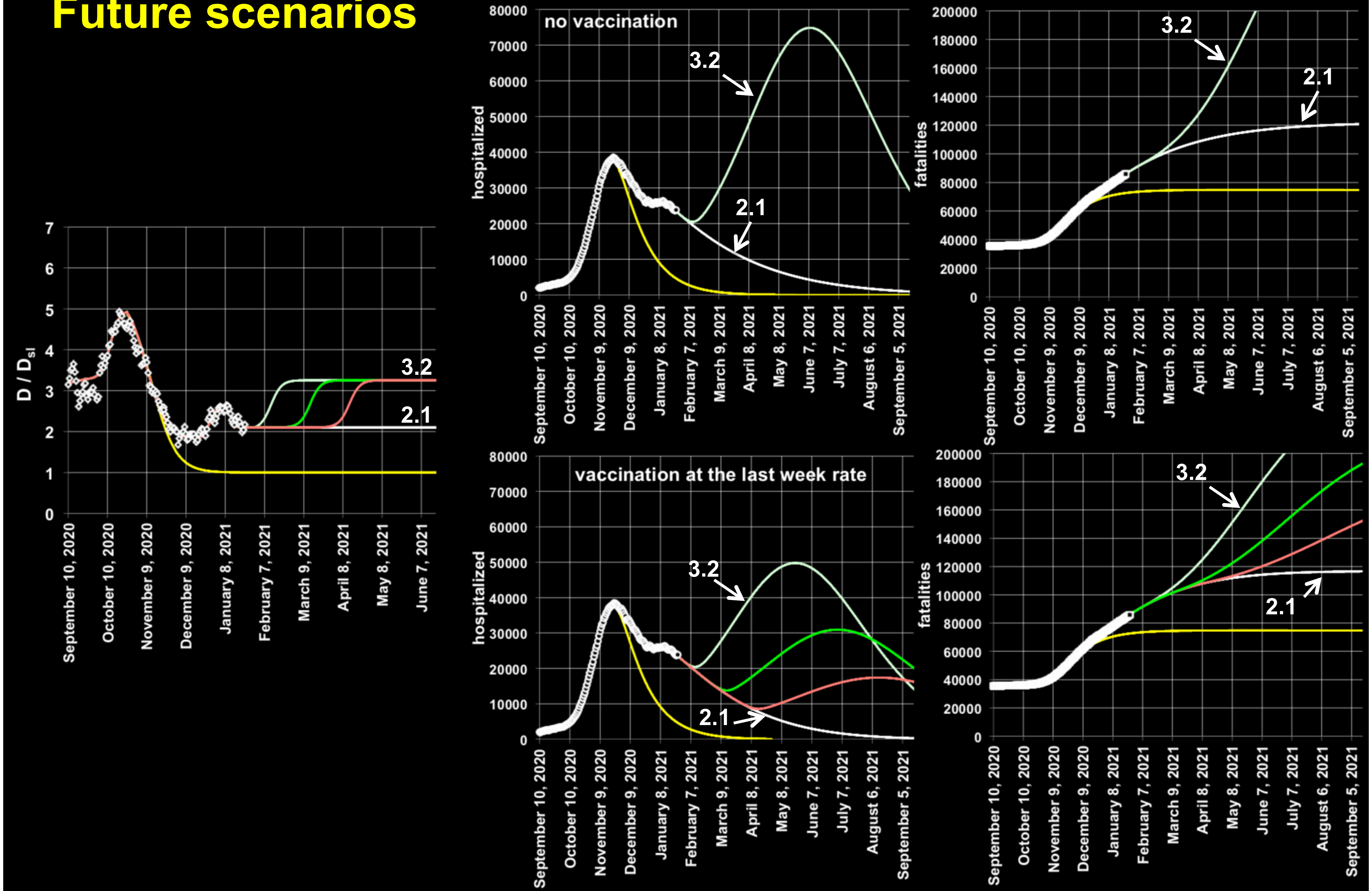
ρ is the number inhabitants diminished by the total number of vaccinated people

c is the number of people who have contracted the virus

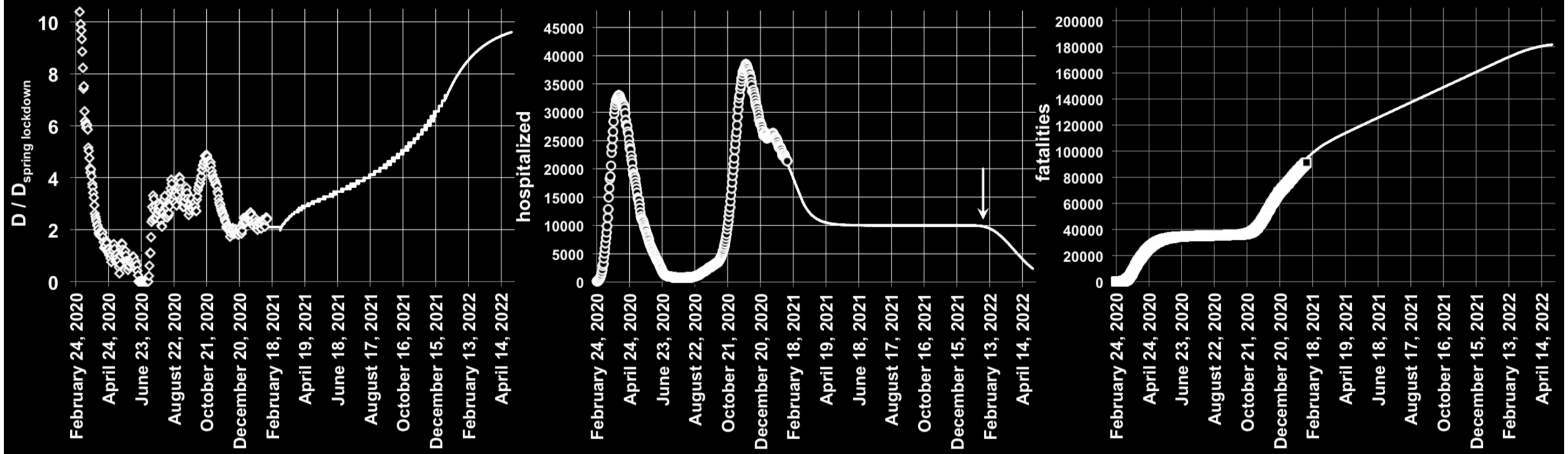
$$t \leq present\ day \quad \Rightarrow \quad v(t) = daily\ reported\ number\ of\ vaccinated\ persons$$

$$t > present\ day \quad \Rightarrow \quad v(t) = a \qquad a = \frac{1}{7} \sum_{day-6}^{day} v(t)$$

Future scenarios



Back to high mobility levels



Evaluation of the reproduction number R_T

$$\frac{1}{\bar{p}_{\theta=0}}\frac{dn}{d\theta} = \rho_0 D\sigma^2[\rho_0 - \rho_i(\theta) - c(\theta)]\,\frac{\bar{p}(\theta)}{\bar{p}_{\theta=0}}$$

n number of cases infected by a concentration $\bar{p}$ probe of positives (that decays with time)

$$P(\theta) = \frac{\bar{p}(\theta)}{\bar{p}_{\theta=0}}$$

$$\rho_i(t) = \int_{t_0+\tau}^{t} v(t-\tau)dt$$

ρ_i number of people immunized by vaccination

$$R_T = \rho_0\sigma^2\int_0^{+\infty} D(\theta)[\rho_0 - \rho_i(\theta) - c(\theta)]P(\theta)d\theta$$

R_T number of people infected by the the positive concentration probe infected by a concentration probe of positives

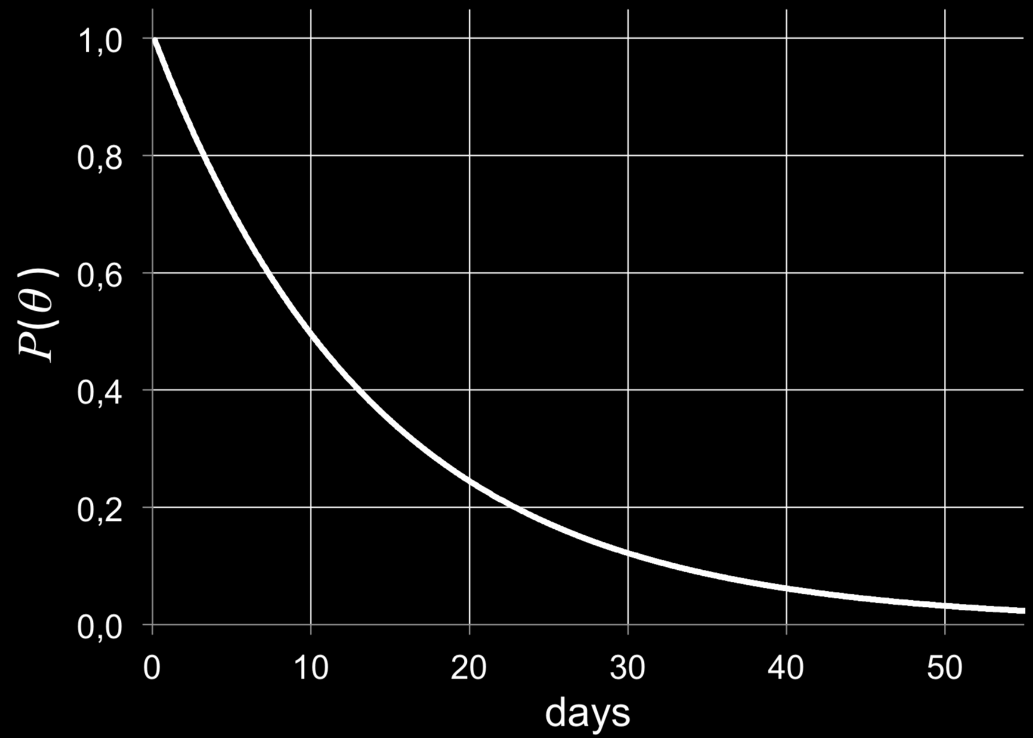
$$T = \frac{\int_0^{+\infty}\theta\frac{dn}{d\theta}d\theta}{\int_0^{+\infty}\frac{dn}{d\theta}d\theta} = \frac{\int_0^{+\infty}\theta D(\theta)[\rho_0 - \rho_i(\theta) - c(\theta)]P(\theta)d\theta}{\int_0^{+\infty}D(\theta)[\rho_0 - \rho_i(\theta) - c(\theta)]P(\theta)d\theta}$$

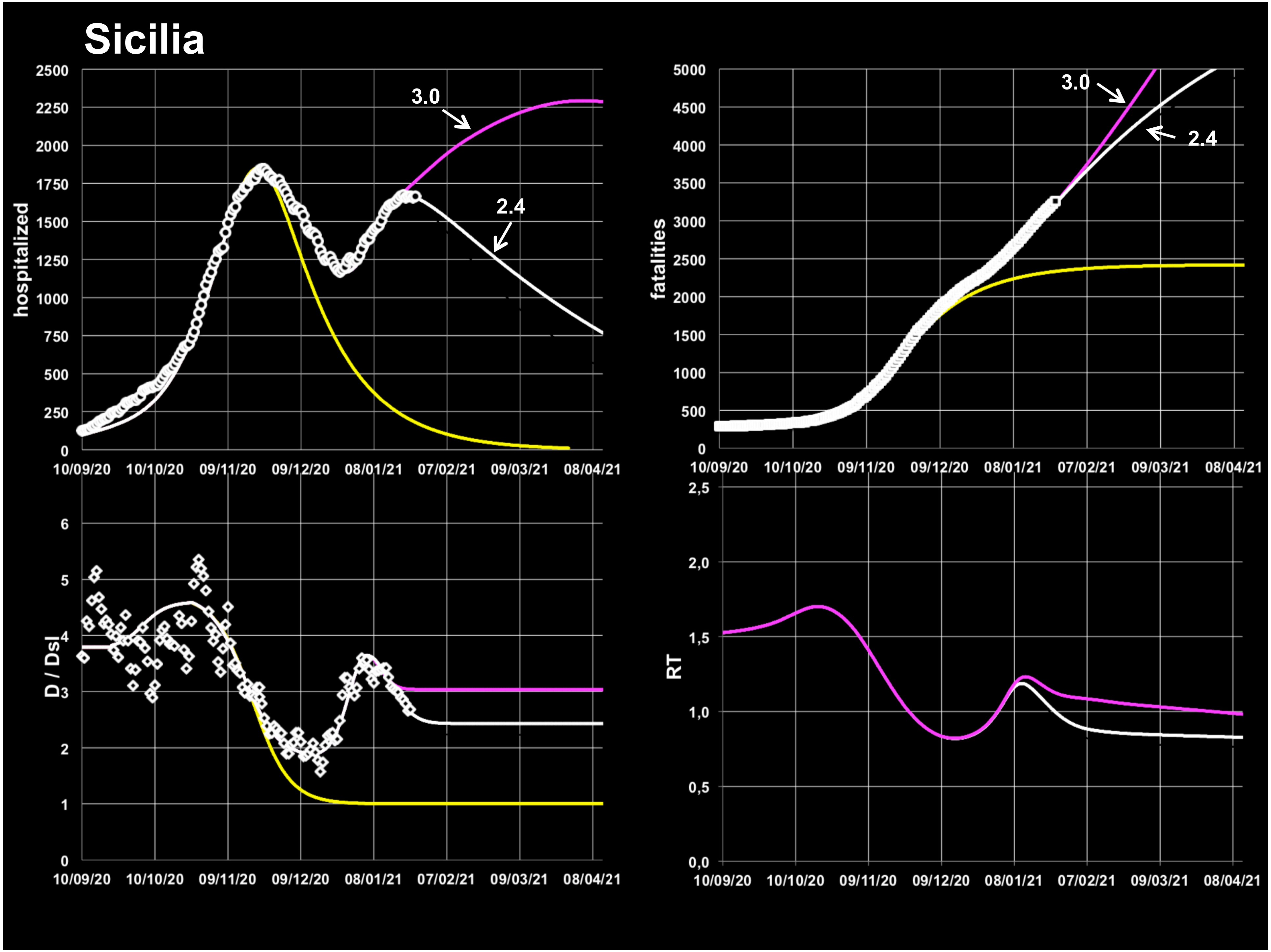
T time the evaluation of R_T is referred to

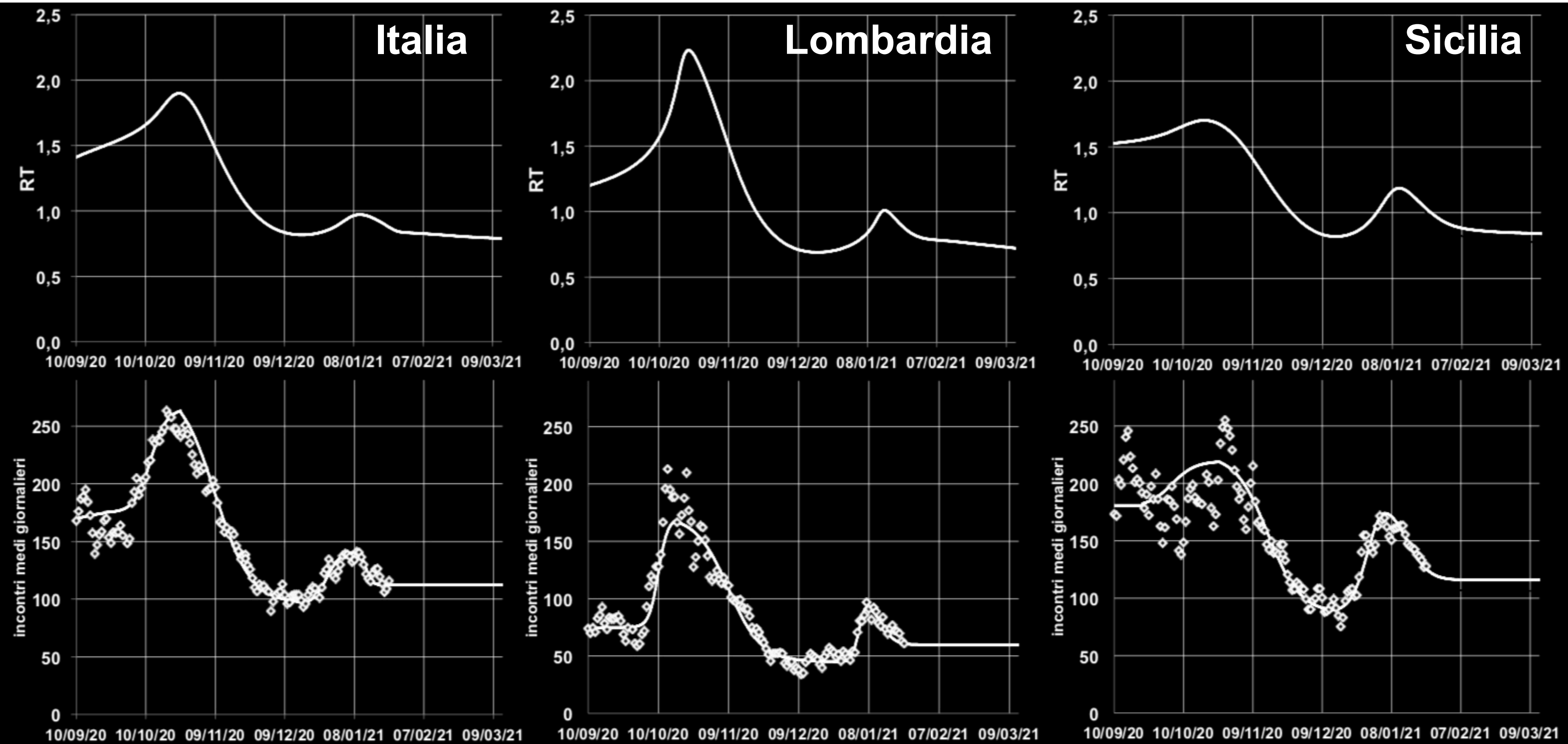
$$\frac{dP}{d\theta} = -\frac{dG}{d\theta} - \frac{dM}{d\theta} \qquad \frac{dR}{d\theta} = -\left(\frac{q}{\tau_1} + \frac{1-q}{\tau_2}\right) R(\theta) \qquad \frac{dM}{d\theta} = \frac{q}{\tau_1} R(\theta)$$

$$\frac{dS}{d\theta} = -\frac{S(\theta)}{\tau_3} \qquad \frac{dG}{d\theta} = \frac{S(\theta)}{\tau_3} + \frac{(1-q)R(\theta)}{\tau_2}$$

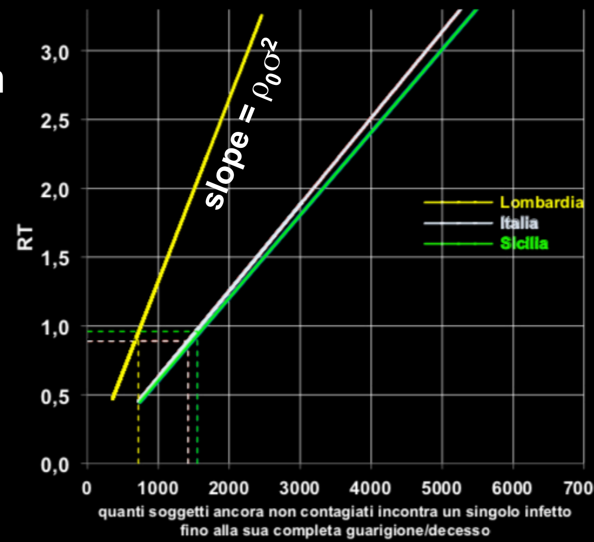
$$P(\theta) \sim (1-f)\exp\left(-\frac{\theta}{\tau_3}\right) + f\exp\left(-\frac{\theta}{\tau_2}\right)$$




Sicilia
hospitalized
2500
2250
2000
1750
1500
1250
1000
750
500
250
0
3.0
2.4
10/09/20
10/10/20
09/11/20
09/12/20
08/01/21
07/02/21
09/03/21
08/04/21
fatalities
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
D / Dsl
6
5
4
3
2
1
0
RT
2,5
2,0
1,5
1,0
0,5
0,0



RT is more sensitive to variation of mobility in Regions with higher density of inhabitants



at a fixed instant we have a concentration p' of positives for a virus variant that can be:

more infective or **more serious**

Impact of virus variants

$\sigma^2 \Rightarrow \sigma'^2$ $\qquad$ $f \Rightarrow f'$

$$\frac{dp}{dt} = \rho_0 D\sigma^2[\rho - c]p - \frac{dg}{dt} - \frac{dm}{dt}$$

$$\frac{dr}{dt} = f\rho D\sigma^2(\rho - c)p - \left(\frac{q}{\tau_1} + \frac{1-q}{\tau_2}\right)r$$

$$\frac{dm}{dt} = \frac{q}{\tau_1}r$$

$$\frac{ds}{dt} = (1-f)\rho D\sigma^2(\rho - c)p - \frac{s}{\tau_3}$$

$$\frac{dg}{dt} = \frac{s}{\tau_3} + \frac{(1-q)r}{\tau_2}$$

$$\frac{dp'}{dt} = \rho_0 D\sigma'^2[\rho - c]p' - \frac{dg'}{dt} - \frac{dm'}{dt}$$

$$\frac{dr'}{dt} = f'\rho D\sigma'^2(\rho - c)p' - \left(\frac{q}{\tau_1} + \frac{1-q}{\tau_2}\right)r'$$

$$\frac{dm'}{dt} = \frac{q'}{\tau_1}r'$$

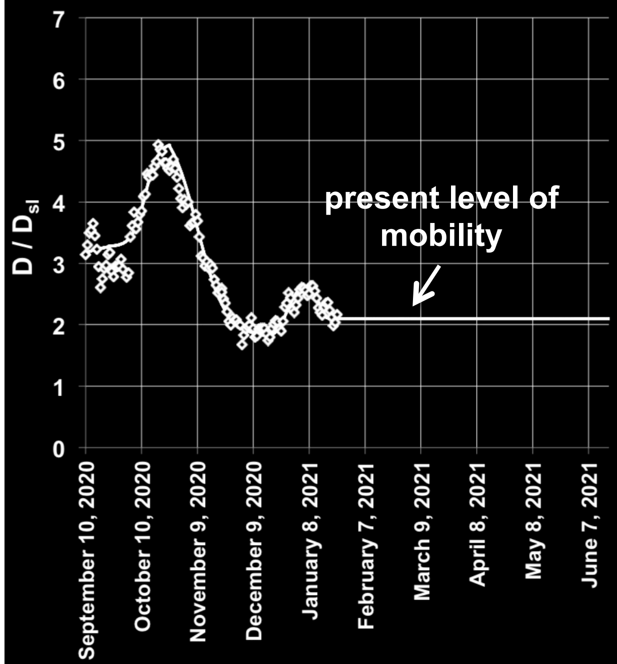
$$\frac{ds'}{dt} = (1-f')\rho D\sigma'^2(\rho - c)p' - \frac{s'}{\tau_3}$$

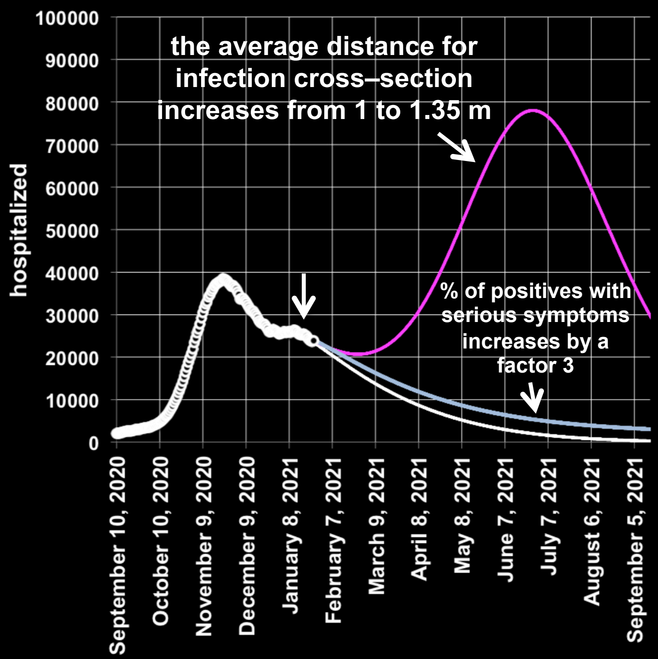
$$\frac{dg'}{dt} = \frac{s'}{\tau_3} + \frac{(1-q)r'}{\tau_2}$$

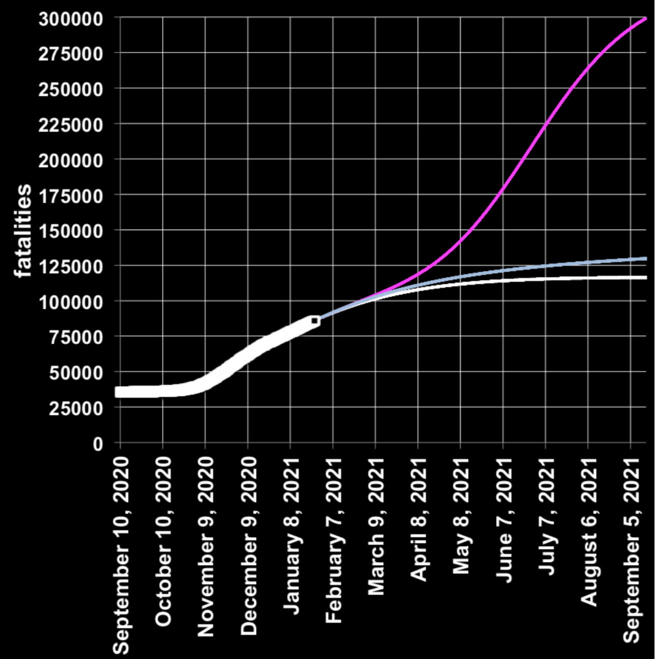
$$\frac{dc}{dt} = \rho D\sigma^2(\rho - c)p + \rho D\sigma'^2(\rho - c)p'$$

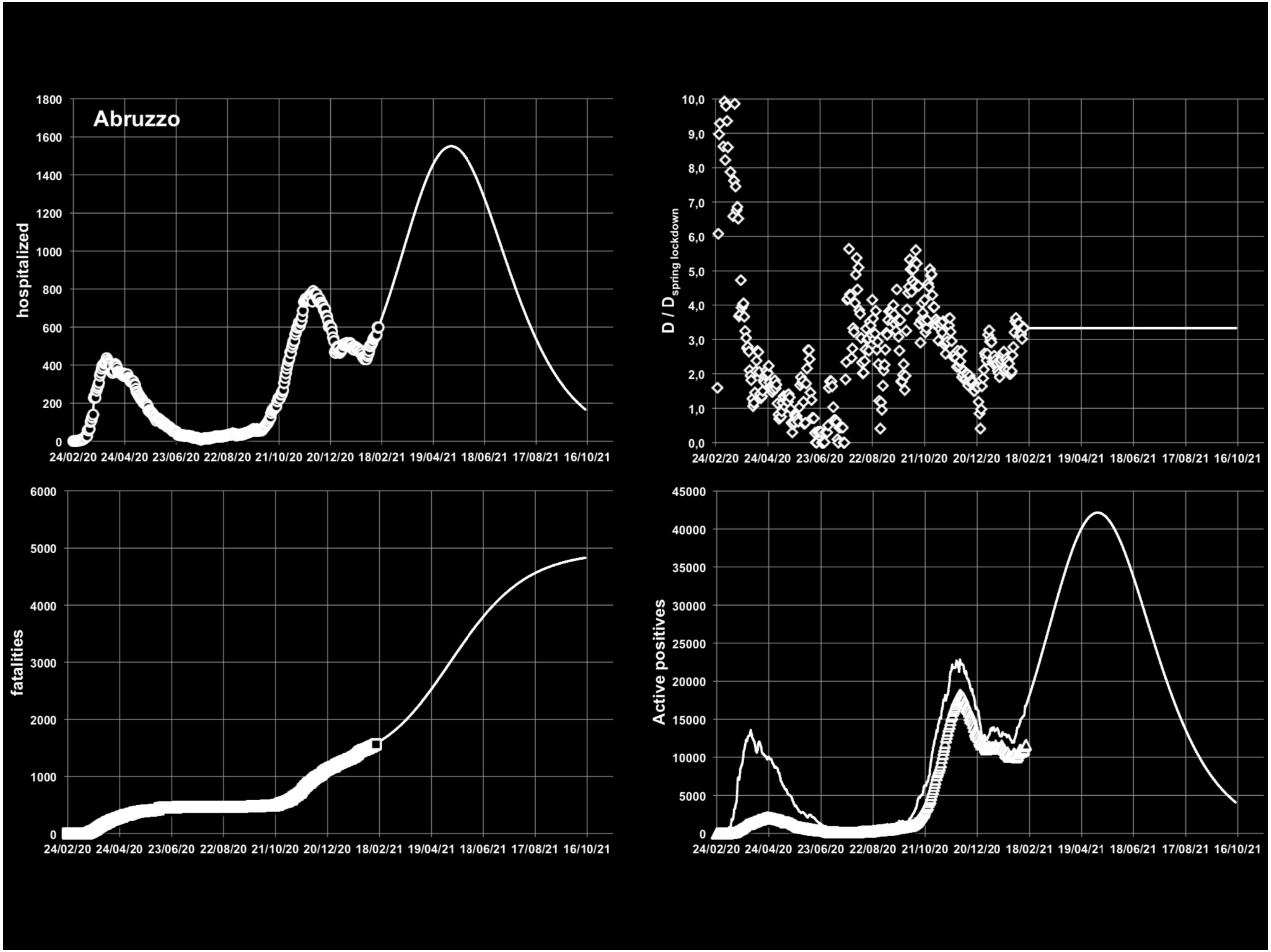
$p + p'$ $\qquad$ $r + r'$

Impact of virus variants

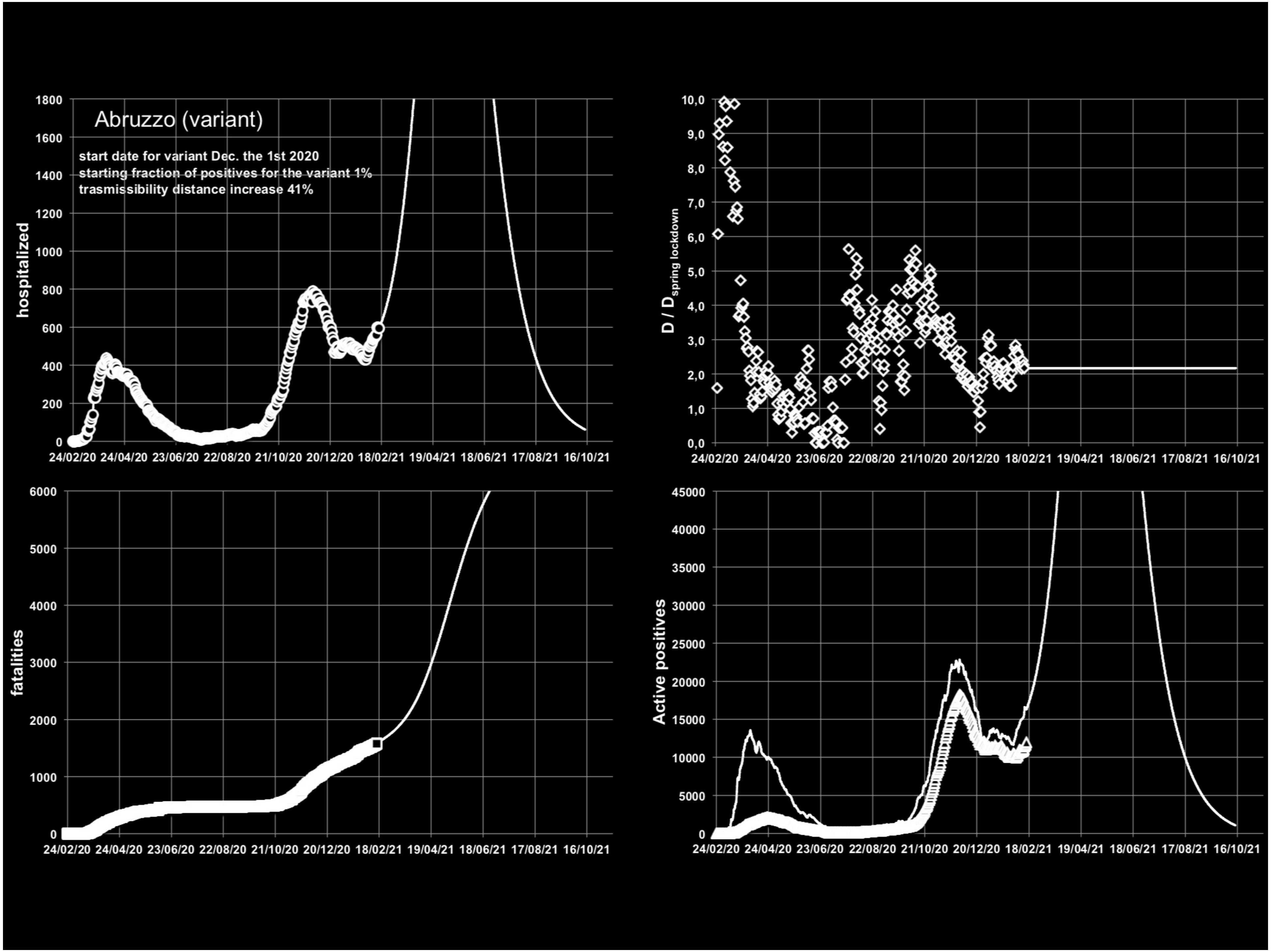






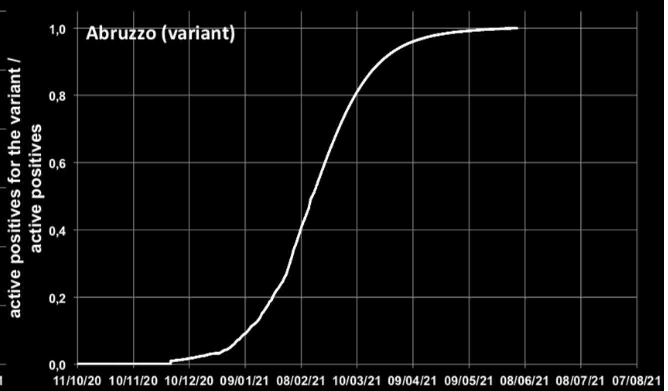


Abruzzo
hospitalized
1800
1600
1400
1200
1000
800
600
400
200
0
24/02/20
24/04/20
23/06/20
22/08/20
21/10/20
20/12/20
18/02/21
19/04/21
18/06/21
17/08/21
16/10/21
D / D spring lockdown
10,0
9,0
8,0
7,0
6,0
5,0
4,0
3,0
2,0
1,0
0,0
fatalities
6000
5000
4000
3000
2000
1000
Active positives
45000
40000
35000
30000
25000
20000
15000
10000
5000


Abruzzo (variant)
start date for variant Dec. the 1st 2020
starting fraction of positives for the variant 1%
trasmissibility distance increase 41%
hospitalized
1800
1600
1400
1200
1000
800
600
400
200
0
24/02/20 24/04/20 23/06/20 22/08/20 21/10/20 20/12/20 18/02/21 19/04/21 18/06/21 17/08/21 16/10/21
D / D spring lockdown
10,0
9,0
8,0
7,0
6,0
5,0
4,0
3,0
2,0
1,0
0,0
24/02/20 24/04/20 23/06/20 22/08/20 21/10/20 20/12/20 18/02/21 19/04/21 18/06/21 17/08/21 16/10/21
fatalities
6000
5000
4000
3000
2000
1000
0
24/02/20 24/04/20 23/06/20 22/08/20 21/10/20 20/12/20 18/02/21 19/04/21 18/06/21 17/08/21 16/10/21
Active positives
45000
40000
35000
30000
25000
20000
15000
10000
5000
0
24/02/20 24/04/20 23/06/20 22/08/20 21/10/20 20/12/20 18/02/21 19/04/21 18/06/21 17/08/21 16/10/21


Abruzzo
RT
3,0
2,5
2,0
1,5
1,0
0,5
0,0
24/02/20 24/04/20 23/06/20 22/08/20 21/10/20 20/12/20 18/02/21 19/04/21 18/06/21 17/08/21 16/10/21


Abruzzo (variant)
RT
3,0
2,5
2,0
1,5
1,0
0,5
0,0
24/02/20 24/04/20 23/06/20 22/08/20 21/10/20 20/12/20 18/02/21 19/04/21 18/06/21 17/08/21 16/10/21


Abruzzo (variant)
active positives for the variant / active positives
1,0
0,8
0,6
0,4
0,2
0,0
11/10/20 10/11/20 10/12/20 09/01/21 08/02/21 10/03/21 09/04/21 09/05/21 08/06/21 08/07/21 07/08/21

Conclusions

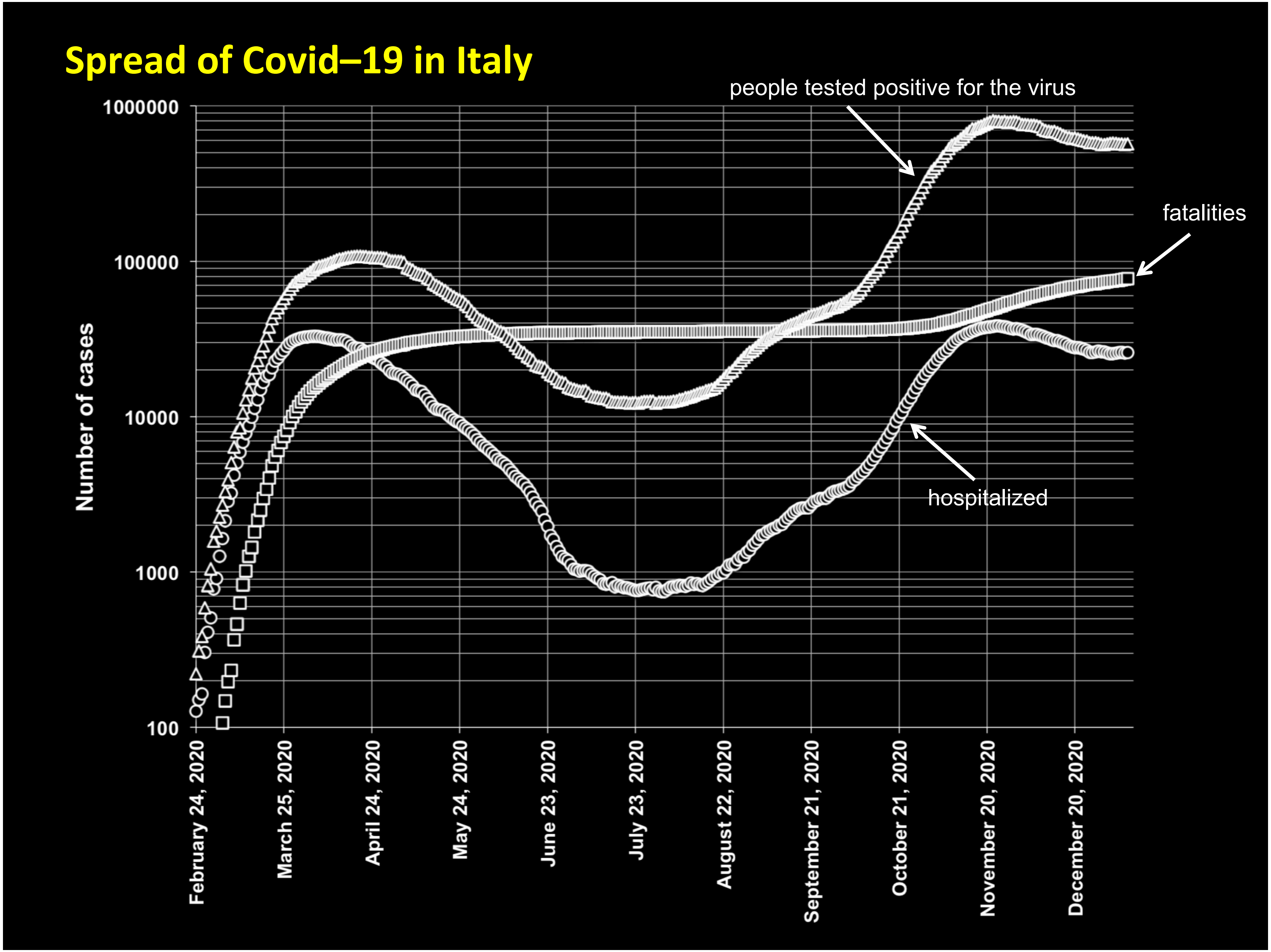
We have presented a phenomenological compartmental model that fits very well the time dependence of hospitalized cases and fatalities during the spread of Covid–19 virus infection in Italy

The model allows us to associate the dynamical evolution of the observed cases to the variation of mobility due to the restriction measures adopted by the Government

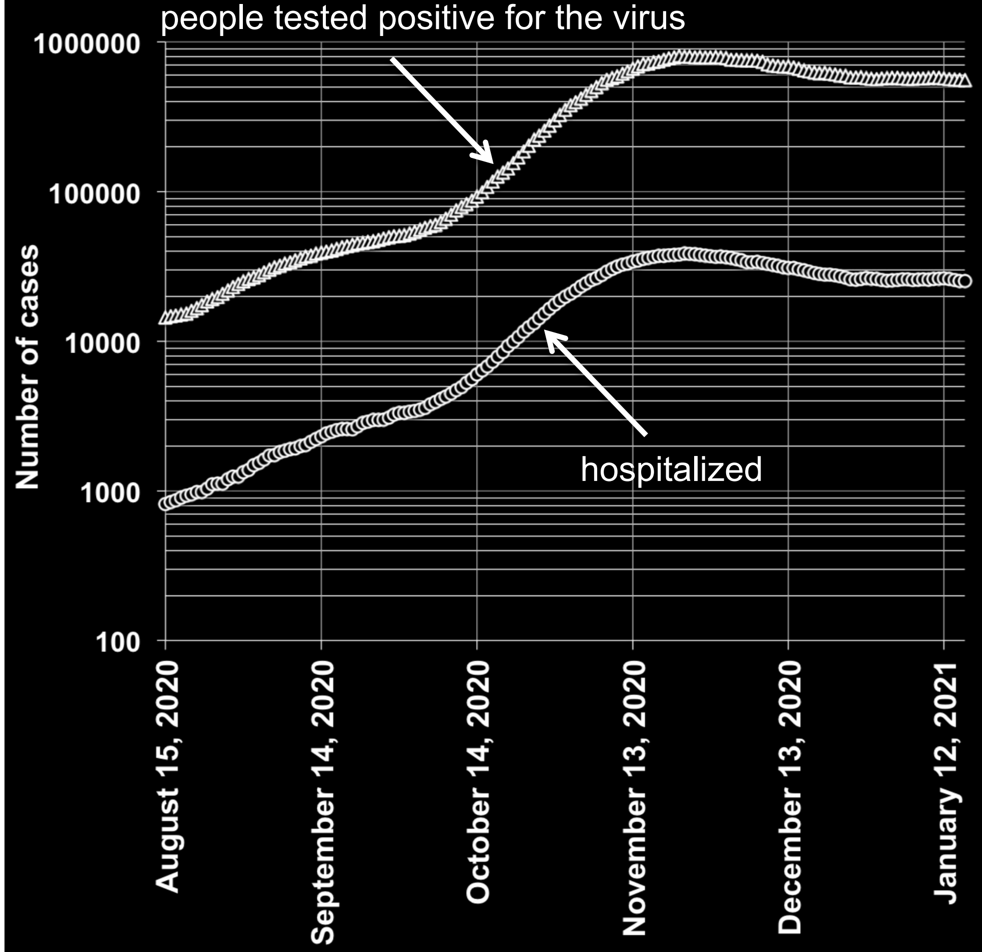
The model can predict future scenarios consequent to the easing of the restriction rules (mobility increase)

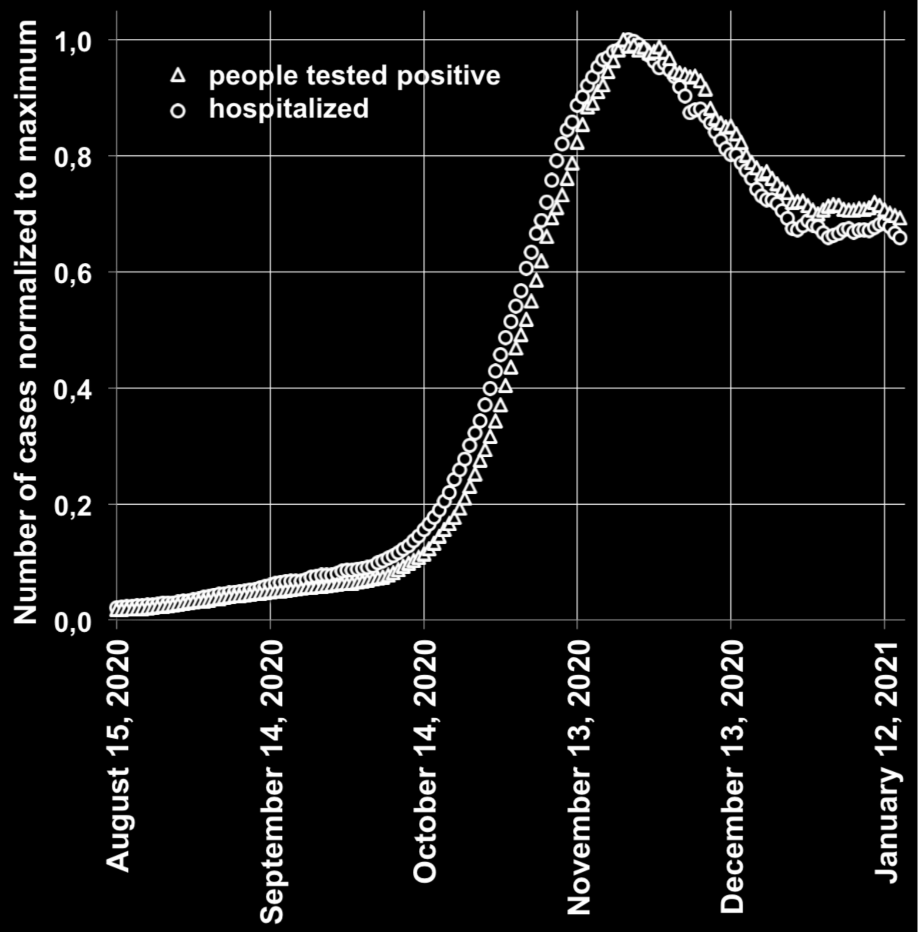
Vaccination, at the present rate, does not allows us to mitigate the containment measures (reduced mobility) in the short period

The impact of virus variants can be modeled by including a further dynamics describing the evolution of new positives for virus with different infection cross–section or severity


Spread of Covid–19 in Italy
people tested positive for the virus
fatalities
hospitalized
Number of cases
1000000
100000
10000
1000
100
February 24, 2020
March 25, 2020
April 24, 2020
May 24, 2020
June 23, 2020
July 23, 2020
August 22, 2020
September 21, 2020
October 21, 2020
November 20, 2020
December 20, 2020

The second wave: the time dependence of the hospitalized cases follows pretty well the one of people tested positive





Hospitalized cases are synchronized to the data on people tested positive for the virus

What happens if we consider a delay time, δ, for those who have serious symptoms, from the instant of the infection to the instant of hospitalization

$$\frac{dp}{dt} = \rho D \sigma^2 (\rho - c) p - \frac{dg}{dt} - \frac{dm}{dt}$$

$$\frac{dw}{dt} = f \rho D \sigma^2 (\rho - c) p - \frac{w}{\delta}$$

$$\frac{dr}{dt} = \frac{w}{\delta} - \left(\frac{q}{\tau_1} + \frac{1-q}{\tau_2}\right) r$$

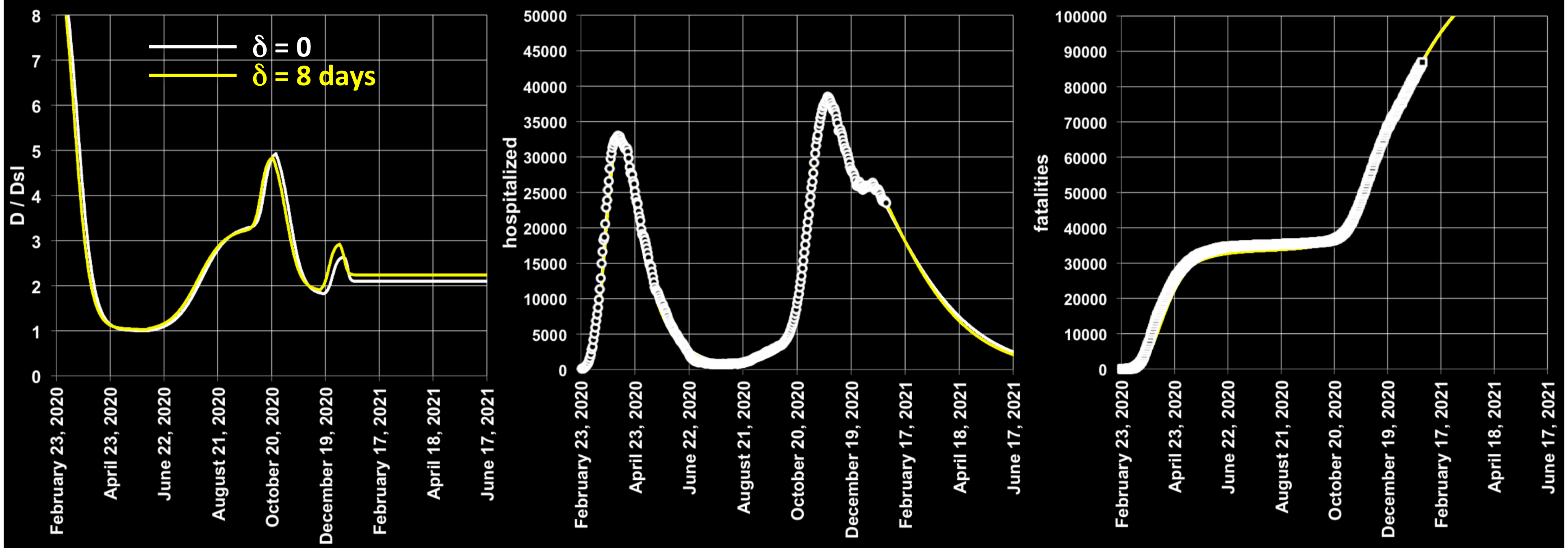
$$\frac{dm}{dt} = \frac{q}{\tau_1} r$$

$$\frac{ds}{dt} = (1-f) \rho D \sigma^2 (\rho - c) p - \frac{s}{\tau_3}$$

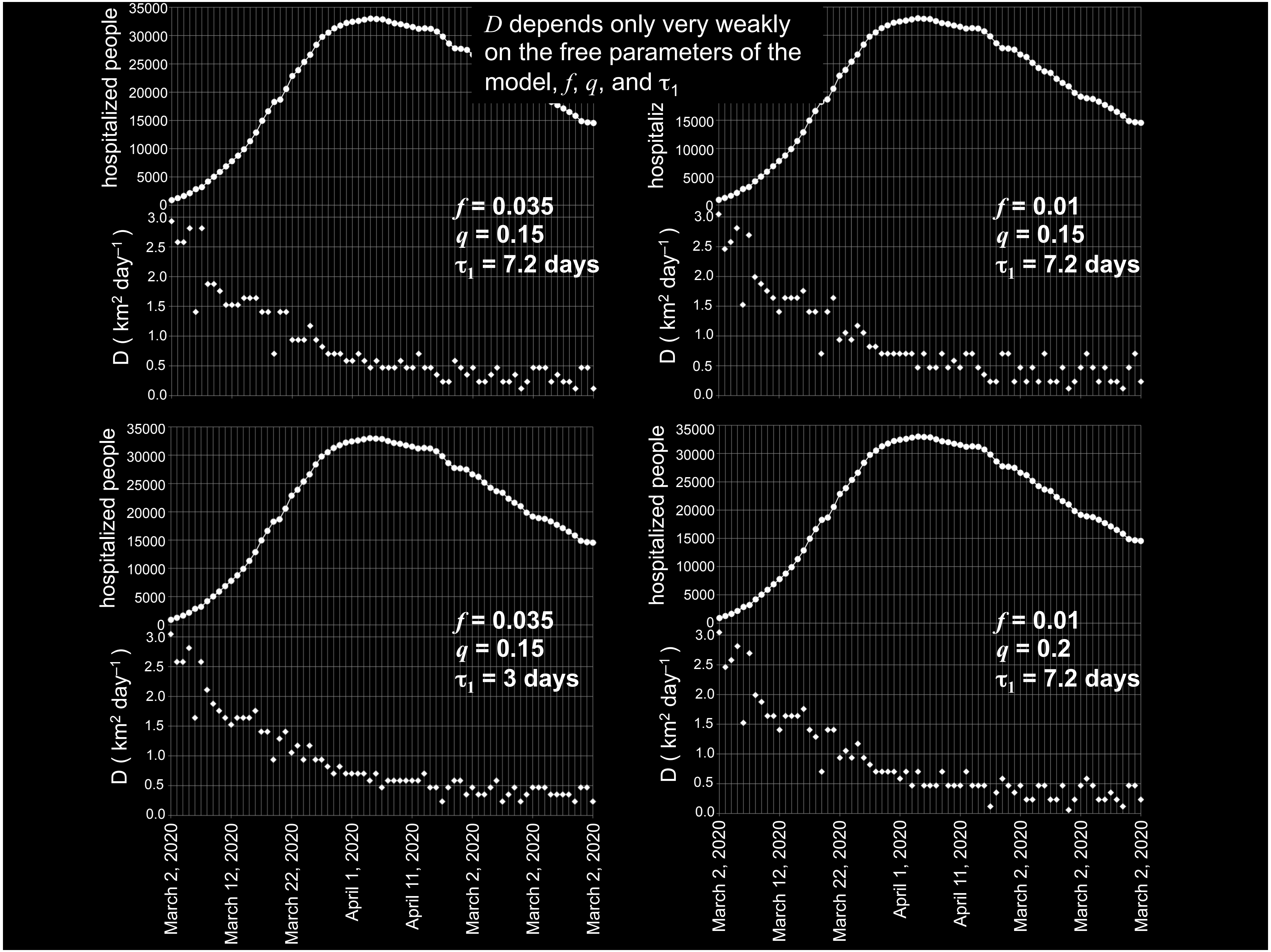
$$\frac{dg}{dt} = \frac{s}{\tau_3} + \frac{(1-q) r}{\tau_2}$$

$$\frac{dc}{dt} = \rho D \sigma^2 (\rho - c) p$$

Hospitalized cases are synchronized to the data on people tested positive for the virus



δ = 0
δ = 8 days
D / DsI
hospitalized
fatalities
February 23, 2020
April 23, 2020
June 22, 2020
August 21, 2020
October 20, 2020
December 19, 2020
February 17, 2021
April 18, 2021
June 17, 2021



D depends only very weakly on the free parameters of the model, f, q, and τ_1
hospitalized people
D (km^2 day^{-1})
f = 0.035
q = 0.15
τ_1 = 7.2 days
f = 0.01
q = 0.15
τ_1 = 7.2 days
f = 0.035
q = 0.15
τ_1 = 3 days
f = 0.01
q = 0.2
τ_1 = 7.2 days
March 2, 2020
March 12, 2020
March 22, 2020
April 1, 2020
April 11, 2020